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Note on Sample Size Determination for Detecting the Worst Component of a Multivariate Exponential Distribution

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1. INTRODUCTION

It is well known that an exponential distribution plays an important role in the field of survival analysis. A multivariate exponential distribution was given by Marshall and Olkin (1967), which is much of interest in both theoretical developments and applications. Especially, Proschan and Sullo (1976) considered a $k+1$-parameter version of the Marshall–Olkin multivariate exponential distribution and studied likelihood estimation of its parameters. We consider a selection problem on components of the $k+1$-parameter exponential distribution.

Let $(X_{1r}, X_{2r}, ..., X_{kr})$, $r = 1, 2, ...$ be random samples from the multivariate exponential (MVE) distribution whose survival function is given by

$$\begin{aligned}&P\left(X_{1r} > x_1,\ X_{2r} > x_2,\ ...,\ X_{kr} > x_k\right)\\&= \exp\left\{-\lambda_1 x_1 - \lambda_2 x_2 - ... - \lambda_k x_k - \lambda_0 \max(x_1, x_2, ..., x_k)\right\}\end{aligned}$$

where $x_i > 0$, $\lambda_i > 0$ $(i = 1, 2, ..., k)$ and $\lambda_0 \geq 0$. Marshall and Olkin (1967) derived this distribution under the assumption that failure is caused by $k+1$ types of Poisson shocks on a system containing k components. We consider the problem of detecting the worst component with respect to λ_i $(i = 1, 2, ..., k)$ which is viewed as the hazard of the component in lifetime analysis. We define that the j-th component is the worst component if $\lambda_j = \max(\lambda_1, \lambda_2, ..., \lambda_k)$. Note that

$$\begin{aligned}&E(X_{ir}) = 1/(\lambda_i + \lambda_0),\\&Var(X_{ir}) = 1/(\lambda_i + \lambda_0)^2,\\&Cov(X_{ir}, X_{jr}) = \lambda_0/(\lambda_i + \lambda_0)(\lambda_j + \lambda_0)(\lambda_i + \lambda_j + \lambda_0),\ i \neq j.\end{aligned}$$

The worst component has the smallest mean. Throughout this paper, we assume that the k-th component is the worst component, that is,

$$\lambda_k > \lambda_i \quad (i = 1, 2, ..., k-1) \tag{1.1}$$

without loss of generality. Since $X_{1r}, X_{2r}, ..., X_{kr}$ are mutually independent if and only if $\lambda_0 = 0$, the problem is reduced to the problem of detecting the worst one out of k univariate exponential populations when $\lambda_0 = 0$.

We seek a procedure R which detects one of the components as the worst component. A correct decision (CD) occurs when the detected component is the worst component. Denoting the probability of a correct decision (PCD) using R by $P(CD|R)$, we require that

$$P(CD|R) \geq P^\star \quad \text{whenever} \quad \lambda_k/\lambda_i \geq \delta^\star \ (i = 1, ..., k-1), \tag{1.2}$$

where $\delta^\star(> 1)$ and $P^\star \in (k^{-1}, 1)$ are specified by the experimenter in advance. The special case of this problem when $k = 2$ was considered by Hyakutake (1992) and Aoshima and Chen (1999). So, in the present paper, we assume that $k \geq 3$. When $k \geq 3$, the former paper by the authors in 2002 has tackled the problem of selecting the best component of the MVE distribution. It would be interesting to observe how much difference the sample size makes between both the situations of selecting the best component and detecting the worst component. Also, from the decision theoretical point of view, this kind of fixed–size estimation on scale parameters is desirable to research in sequential analysis especially based on a two–stage sampling scheme since it must deal with a bit outside Stein's original framework. This is a motivation of this short note. Note that detecting the worst component with respect to λ_i is equivalent to that with respect to $p_i = \lambda_i / \sum_{\ell=0}^{k} \lambda_\ell$. Then, we have from Arnold (1968) that

$$\begin{aligned} &P(X_{1r} = X_{2r} = ... = X_{kr}) = p_0, \\ &P(X_{ir} < X_{i'r},\ i' = 1, ..., i-1, i+1, ..., k) = p_i,\ i = 1, 2, ..., k. \end{aligned} \tag{1.3}$$

The preference zone $\lambda_k/\lambda_i \geq \delta^\star$ $(i = 1, ..., k-1)$ in (1.2) is equivalent to $p_k/p_i \geq \delta^\star$ $(i = 1, ..., k-1)$.

2. SINGLE–STAGE PROCEDURES WHEN p_0 IS KNOWN

In this section, we assume that p_0 is known and consider two single–stage procedures $(R_\ell,\ \ell = 1, 2)$ for detecting the component associated with p_k. In order to meet requirement (1.2), we need to establish the least favorable configuration (LFC) of $\boldsymbol{\lambda} = (\lambda_0, \lambda_1, ..., \lambda_k)$ for which the PCD for R_ℓ is minimized subject to the condition that $\lambda_k/\lambda_i \geq \delta^\star$ $(i = 1, ..., k-1)$. The LFC for a procedure would be given by

$$\lambda_0,\ \lambda_1 = ... = \lambda_{k-1} = \lambda_k/\delta^\star \quad (\delta^\star > 1). \tag{2.1}$$

We shall evaluate the PCD for R_ℓ under the LFC given by (2.1), which we will denote by $P^\star(CD|R_\ell)$. Under the LFC, we note that

$$\begin{aligned} p_1 &= \dots = p_{k-1} = p_k/\delta^\star, \\ p_k &= \delta^\star(1-p_0)/(\delta^\star + k - 1). \end{aligned} \tag{2.2}$$

2.1 Procedure R_1

We consider the following rule (R_1): Take a sample of n observations. Let n_i and n_0 denote the number of observations in the regions $\{x_i < x_{i'}\}$ and $\{x_1 = \dots = x_k\}$, respectively. Detect the component that attained the largest count among $(n_1, \dots, n_k)$ as the worst component; use randomization to break ties for the first place.

We note that $(n_1, \dots, n_k, n_0)$ has a multinomial distribution with $n = \sum_{\ell=0}^{k} n_\ell$ and $k+1$ cell probabilities $(p_1, \dots, p_k, p_0)$ in view of (1.3). Then, the rule R_1 is exactly reduced to the rule R given by Aoshima, Chen and Panchapakesan (2001) for selecting the most probable cell where n_0 is the count in the nuisance cell. When n is large, they gave an approximate minimum sample size needed to satisfy requirement (1.2) as $n_0^{(1)} = [u_1^2/c_1^2] + 1$, where $[x]$ denotes the greatest integer less than x. Here, $u_1 > 0$ is $u_1 = u_1(p_0, k, \delta^\star, P^\star)$ such that

$$\int_{-\infty}^{\infty} \Phi^{k-1}\left(\frac{x\sqrt{\rho_1} + u_1}{\sqrt{1-\rho_1}}\right) d\Phi(x) = P^\star$$

with

$$\rho_1 = \frac{p_0(\delta^\star - 1)^2 + (1+k)\delta^\star - 1}{p_0(\delta^\star - 1)^2 + \delta^\star(2+k) - 2 + k},$$

and

$$c_1 = \left\{\frac{(1-p_0)(\delta^\star - 1)^2}{p_0(\delta^\star - 1)^2 + \delta^\star(2+k) - 2 + k}\right\}^{1/2} = c_1(p_0, k, \delta^\star), \text{ (say)}. \tag{2.3}$$

Since u_1 depends on p_0 through ρ_1, they gave an upper bound for $n_0^{(1)}$ as $n_1^{(1)} = [u_1^{\star 2}/c_1^2] + 1$ where $u_1^\star > 0$ is $u_1^\star = u_1^\star(k, \delta^\star, P^\star)$ such that

$$\int_{-\infty}^{\infty} \Phi^{k-1}\left(\frac{x\sqrt{\rho_1^\star} + u_1^\star}{\sqrt{1-\rho_1^\star}}\right) d\Phi(x) = P^\star \text{ with } \rho_1^\star = \frac{-1 + \delta^\star + k\delta^\star}{-2 + k + 2\delta^\star + k\delta^\star}.$$

The following table was given by Aoshima, Chen and Panchapakesan (2001).

Table 2.1. Values of $u_1^\star$ when $P^\star = 0.90$

$\delta^\star \setminus k$	3	4	5	6	7	8	9	10
1.5	1.559	1.704	1.801	1.872	1.929	1.975	2.015	2.049
2.0	1.547	1.684	1.774	1.841	1.893	1.936	1.972	2.004
2.5	1.539	1.670	1.755	1.817	1.866	1.906	1.940	1.969
3.0	1.533	1.659	1.740	1.799	1.845	1.882	1.914	1.941

When p_0 is unknown, we need to estimate $n_1^{(1)}$ through an estimate $\widehat{p}_0$ based on a pilot sample.

2.2 Procedure R_2

If a complete data set (without cesoring) is available, we can give another rule (R_2) which is based on the marginal distribution. The distribution of X_{ir} is the exponential distribution with parameter $\lambda_i+\lambda_0$. The selection problem with respect to λ_i is equivalent to that to $\lambda_i+\lambda_0$. Let $\overline{X}_{in}=\sum_{r=1}^{n} X_{ir}/n$, then $\overline{X}_{in}$ is an unbiased estimate of $1/(\lambda_i+\lambda_0)$. Proschan and Sullo (1976) showed that $\overline{X}_{in}$ is also the INT estimator. Then, the rule R_2 is described: Take a sample of n observations. Detect the component associated with $\overline{X}_{jn}=\min(\overline{X}_{1n},...,\overline{X}_{kn})$ as the worst component.

It is easy to see that under the LFC, the PCD is written for n large that

$$\begin{aligned} P^\star(CD|R_2) &= P(\overline{X}_{kn}<\overline{X}_{in},\ i=1,...,k-1) \\ &\simeq \int_{-\infty}^{\infty} \Phi^{k-1}\left(\frac{x\sqrt{\rho_2}+c_2\sqrt{n}}{\sqrt{1-\rho_2}}\right) d\Phi(x), \end{aligned} \tag{2.4}$$

where $\rho_2=\rho_2(p_0,k,\delta^\star)>0$ is defined as

$$\begin{aligned} \rho_2 = \Big\{ & 2(1+\delta^\star)+p_0(-13+\delta^{\star 4}+k(7+\delta^\star+\delta^{\star 2}+\delta^{\star 3})) \\ & +p_0^2(25-16\delta^\star+8\delta^{\star 2}-6\delta^{\star 3}+\delta^{\star 4}+4k(-6+2\delta^\star-2\delta^{\star 2}+\delta^{\star 3})+k^2(5+3\delta^{\star 2})) \\ & +p_0^3(-2(7-6\delta^\star+\delta^{\star 2})+k(17-3\delta^\star-5\delta^{\star 2}+\delta^{\star 3})+k^2(-5-6\delta^\star+3\delta^{\star 2})+2k^3\delta^\star)\Big\} \\ & (2-3p_0+kp_0+\delta^\star p_0)^{-1}(1+\delta^\star-3p_0+2kp_0+\delta^\star p_0)^{-1}(1+\delta^{\star 2}-2p_0+kp_0+k\delta^\star p_0)^{-1} \end{aligned}$$

and

$$\begin{aligned} c_2 &= \left\{\frac{(1-p_0)(\delta^\star-1)^2(1+\delta^\star-2p_0+kp_0)}{(1+\delta^\star-3p_0+2kp_0+\delta^\star p_0)(1+\delta^{\star 2}-2p_0+kp_0+k\delta^\star p_0)}\right\}^{1/2} \\ &= c_2(p_0,k,\delta^\star), \text{ (say).} \end{aligned} \tag{2.5}$$

So, if we solve for $u_2=u_2(p_0,k,\delta^\star,P^\star)>0$ from the equation

$$\int_{-\infty}^{\infty} \Phi^{k-1}\left(\frac{x\sqrt{\rho_2}+u_2}{\sqrt{1-\rho_2}}\right) d\Phi(x)=P^\star,$$

an approximate minimum sample size needed to satisfy requirement (1.2) is given by $n_0^{(2)}=[u_2^2/c_2^2]+1$.

Since u_2 depends on p_0 through ρ_2, we consider some upper bounds for $n_0^{(2)}$. Noting that $\rho_2=\rho_2(p_0,k,\delta^\star)$ is increasing in p_0 for any fixed $\delta^\star>1$, we have

$$\rho_2=\rho_2(p_0,k,\delta^\star)>\rho_2(0,k,\delta^\star)=\frac{1}{1+\delta^{\star 2}}=\rho_2^\star \text{ (say).}$$

By using Slepian's inequality, the integral in (2.4) is bounded below by

$$\int_{-\infty}^{\infty} \Phi^{k-1}\left(\frac{x\sqrt{\rho_2^\star}+c_2\sqrt{n}}{\sqrt{1-\rho_2^\star}}\right) d\Phi(x).$$

Then, we have an upper bound for $n_0^{(2)}$ as $n_1^{(2)} = [u_2^{\star 2}/c_2^2]+1$ where $u_2^\star > 0$ is given freely from p_0 by solving the equation

$$\int_{-\infty}^{\infty} \Phi^{k-1}\left(\frac{x\sqrt{\rho_2^\star}+u_2^\star}{\sqrt{1-\rho_2^\star}}\right) d\Phi(x) = P^\star.$$

The following table gives values of $u_2^\star$ for $k = 3(1)10$ and $\delta^\star = 1.5(0.5)3.0$ when $P^\star = 0.90$.

Table 2.2. Values of $u_2^\star$ when $P^\star = 0.90$

$\delta^\star \setminus k$	3	4	5	6	7	8	9	10
1.5	1.606	1.779	1.895	1.981	2.050	2.107	2.155	2.197
2.0	1.617	1.796	1.917	2.007	2.078	2.137	2.187	2.230
2.5	1.623	1.805	1.927	2.018	2.090	2.150	2.201	2.246
3.0	1.626	1.809	1.932	2.024	2.097	2.157	2.209	2.253

When p_0 is unknown, we need to estimate $n_1^{(2)}$ through an estimate $\widehat{p}_0$ based on a pilot sample.

Remark 1. When we note that $\delta^\star \geq \rho_2(1-\rho_2)^{-1} \geq \delta^{\star -2}$ in (2.4), another upper bound for $n_0^{(2)}$ is obtained as $n_2^{(2)} = [(1-\rho_2)\tilde{u}_2^2/c_2^2]+1$ where $\tilde{u}_2 > 0$ is given freely from p_0 by solving the equation

$$\int_{-\infty}^{0} \Phi^{k-1}\big(x\sqrt{\delta^\star}+\tilde{u}_2\big)d\Phi(x) + \int_{0}^{\infty} \Phi^{k-1}\left(\frac{x}{\delta^\star}+\tilde{u}_2\right) d\Phi(x) = P^\star.$$

3. TWO–STAGE PROCEDURES

Before considering a two–stage procedure for dealing with the case when p_0 is unknown, let us compare the efficiencies of the procedures R_1 and R_2 in terms of the required sample size for given p_0. We calculated the values of the ratios $n_1^{(1)}/n_1^{(2)}$ and $n_1^{(1)}/n_2^{(2)}$ for $p_0 = 0.1(0.1)0.9$ when $k = 3(1)10$, $\delta^\star = 1.5(0.5)3.0$ and $P^\star = 0.90$. The findings of such survey were as follows: The ratio $n_1^{(1)}/n_1^{(2)}$ is increasing in k for $p_0 < 0.4$, decreasing in k for $p_0 \geq 0.5$ and decreasing in $\delta^\star$ and p_0. The ratio $n_1^{(1)}/n_2^{(2)}$ is increasing in k and decreasing in $\delta^\star$ and p_0. When $\delta^\star = 1.5$, $n_1^{(1)}/n_1^{(2)} > 1$ for $p_0 \leq 0.4$; then $n_1^{(1)}/n_1^{(2)} > n_1^{(1)}/n_2^{(2)}$ except for the cases that $k = 9$ and 10 for $p_0 = 0.4$. When $\delta^\star = 2.0$ and 2.5, $n_1^{(1)}/n_1^{(2)} > 1$ for $p_0 \leq 0.3$ except for the cases that $k = 3$ and 4 for $p_0 = 0.3$ when $\delta^\star = 2.5$; then

$n_1^{(1)}/n_1^{(2)} > n_1^{(1)}/n_2^{(2)}$. When $\delta^\star = 3.0$, $n_1^{(1)}/n_1^{(2)} > 1$ for $p_0 \le 0.2$; then $n_1^{(1)}/n_1^{(2)} > n_1^{(1)}/n_2^{(2)}$. Consequently, when p_0 is small (say, $p_0 \le 0.4$ for $\delta^\star = 1.5$, $p_0 \le 0.3$ for $\delta^\star = 2.0$ and 2.5, and $p_0 \le 0.2$ for $\delta^\star = 3.0$), the procedure R_2 with $n_1^{(2)}$ seems to be the most preferable for consideration to start thinking a two–stage procedure. Then, for other cases, that is p_0 is moderate or large, the procedure R_1 with $n_1^{(1)}$ might be considered. We will compare R_2 with R_1 later by estimating p_0 in the two–stage procedure described below.

For Procedure R_1

We apply one of the procedures given by Aoshima, Chen and Panchapakesan (2001), which is to select the most probable cell of a multinomial distribution in the presence of a nuisance cell, for the present problem. The following procedure (S_1) is based on $n_1^{(1)}$: First, take a sample of size m, which is moderately large. Let m_0 denote the count of $\{x_1 = \ldots = x_k\}$. Compute $\widehat{c}_1 = c_1(\widehat{p}_0, k, \delta^\star)$ with $\widehat{p}_0 = m_0/m$ as in (2.3). Define $N^{(1)}$ by

$$N^{(1)} = \max\left\{m, \left[u_1^{\star 2}/\widehat{c}_1^2\right] + 1\right\}. \tag{3.1}$$

Next, take an additional sample of size $N^{(1)} - m$. On the basis of the total sample of size $N^{(1)}$, let $N_i^{(1)}$ be the count of $\{x_i < x_{i'}\}$ for $i = 1, \ldots, k$. Then, detect the component that attained the largest count among $(N_1^{(1)}, N_2^{(1)}, \ldots, N_k^{(1)})$ as the worst component; use randomization to break ties for the first place.

When m is moderately large, it is shown that Procedure S_1 satisfies requirement (1.2).

For Procedure R_2

We propose the following two–stage procedure (S_2) based on $n_1^{(2)}$: First, take a sample of size m, which is moderately large. Let m_0 denote the count of $\{x_1 = \ldots = x_k\}$. Compute $\widehat{c}_2 = c_2(\widehat{p}_0, k, \delta^\star)$ with $\widehat{p}_0 = m_0/m$ as in (2.5). Define $N^{(2)}$ by

$$N^{(2)} = \max\left\{m, \left[u_2^{\star 2}/\widehat{c}_2^2\right] + 1\right\}. \tag{3.2}$$

Next, take an additional sample of size $N^{(2)} - m$. Then, on the basis of the total sample of size $N^{(2)}$, detect the component associated with the smallest sample mean among $(\overline{X}_{1N^{(2)}}, \overline{X}_{2N^{(2)}}, \ldots, \overline{X}_{kN^{(2)}})$ as the worst component.

It can be shown that Procedure S_2 also satisfies requirement (1.2) when m is moderately large.

Now, let us investigate into efficincies of Procedures S_1 and S_2 through several simulation studies. We estimated the PCS and the expected sample size by conducting the simulation with 10,000 (= R, say) trials for each procedure. The following result given by

Proschan and Sullo (1976, Thorem 2.1) was used for generation of k-variate exponential random number: Let $U_0, U_1, ..., U_k$ be independent exponential random numbers with parameters $\lambda_0, \lambda_1, ..., \lambda_k$, respectively. Then, $(T_1, ..., T_k) = (\min(U_1, U_0), ..., \min(U_k, U_0))$ has the k-variate exponential distribution with parameters $(\lambda_0, \lambda_1, ..., \lambda_k)$. Now, we set $k = 4$ and $\lambda_4 = 12$. Then, under the LFC, the parameters are written as $(\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = \left(\frac{12p_0(3+\delta^*)}{\delta^*(1-p_0)},\ 12/\delta^*,\ 12/\delta^*,\ 12/\delta^*,\ 12\right)$. Set $\delta^* = 1.5, 2.0$ and $P^* = 0.90$. Then, for every case, we used the values of u_1^* and u_2^* respectively given in Tables 2.1 and 2.2. We started S_1 and S_2 with a pilot sample of size $m = [\ 0.8\min\{n_0^{(1)}, n_0^{(2)}\}\ ] + 1$ for $p_0 = 0.1(0.1)0.9$, where $n_0^{(1)}$ and $n_0^{(2)}$ are considered as optimal fixed–sample sizes.

For each procedure S, let us write n_0 as n^*. Let n_r be the observed value of N and $p_r = 1$ (or 0) according as a correct decision occurs (or does not occur). We denote $\overline{n} = \sum_{r=1}^{R} n_r/R$, $s^2(\overline{n}) = \sum_{r=1}^{R}(n_r - \overline{n})^2/(R^2 - R)$, $\overline{p} = \sum_{r=1}^{R} p_r/R$ and $s^2(\overline{p}) = \overline{p}(1-\overline{p})/R$. The quantities $\overline{n}$ and $\overline{p}$ respectively estimate $E(N)$ and $P(CD)$, while $s(\overline{n})$ and $s(\overline{p})$ stand for their corresponding estimated standard errors. In Table 3.1, we report the values of p_0, n^*, m, $\overline{p}$, $s(\overline{p})$, $\overline{n}$, $s(\overline{n})$ and $(\overline{n} - n^*)/n^*$ for each δ^*. For each p_0, the upper line gives those values for S_1 and the lower line gives for S_2.

Table 3.1. Estimated $P(CD)$ and $E(N)$ for S_1 (the upper) and S_2 (the lower) with 10,000 trials when $P^* = 0.90$ and $k = 4$

$\underline{\delta^* = 1.5}$

p_0	n^*	m	$\overline{p}$	$s(\overline{p})$	$\overline{n}$	$s(\overline{n})$	$(\overline{n} - n^*)/n^*$
0.1	142.2	53	0.9066	0.0029	143.1	0.0693	0.0062
	65.17		0.9081	0.0029	67.72	0.1259	0.0392
0.2	160.3	78	0.9087	0.0029	161.3	0.0945	0.0061
	97.15		0.9118	0.0028	102.4	0.1852	0.0545
0.3	183.6	112	0.9125	0.0028	185.2	0.1201	0.0089
	140.0		0.9165	0.0028	149.1	0.2409	0.0647
0.4	214.6	160	0.9061	0.0029	216.4	0.1427	0.0087
	198.9		0.9195	0.0027	213.1	0.2962	0.0716
0.5	258.0	207	0.9088	0.0029	260.4	0.1852	0.0097
	282.9		0.9159	0.0028	305.2	0.3915	0.0791
0.6	323.0	259	0.9102	0.0029	326.6	0.2553	0.0111
	410.1		0.9149	0.0028	446.0	0.5487	0.0864
0.7	431.5	346	0.9138	0.0028	436.8	0.3638	0.0123
	625.3		0.9114	0.0028	682.5	0.7997	0.0915
0.8	648.5	519	0.9071	0.0029	656.4	0.5948	0.0122
	1057		0.9171	0.0028	1159	1.288	0.0958
0.9	1299	1040	0.9123	0.0028	1314	1.256	0.0113
	2357		0.9089	0.0029	2595	2.713	0.1007

$\delta^\star = 2.0$

p_0	$n^\star$	m	$\overline{p}$	$s(\overline{p})$	$\overline{n}$	$s(\overline{n})$	$(\overline{n}-n^\star)/n^\star$
0.1	44.38	20	0.9106	0.0029	45.25	0.0395	0.0194
	24.23		0.9311	0.0025	26.15	0.0730	0.0792
0.2	50.22	28	0.9121	0.0028	51.16	0.0550	0.0187
	34.81		0.9236	0.0027	38.17	0.1102	0.0965
0.3	57.73	40	0.9129	0.0028	59.14	0.0685	0.0243
	48.84		0.9251	0.0026	54.51	0.1427	0.1161
0.4	67.74	55	0.9154	0.0028	69.58	0.0821	0.0272
	67.99		0.9285	0.0026	76.74	0.1818	0.1288
0.5	81.76	66	0.9172	0.0028	84.22	0.1105	0.0301
	95.22		0.9270	0.0026	108.4	0.2488	0.1379
0.6	102.8	83	0.9222	0.0027	106.3	0.1541	0.0340
	136.6		0.9263	0.0026	156.6	0.3452	0.1470
0.7	137.8	111	0.9180	0.0027	142.7	0.2226	0.0352
	206.0		0.9250	0.0026	239.3	0.5014	0.1619
0.8	207.9	167	0.9196	0.0027	215.9	0.3607	0.0388
	345.5		0.9193	0.0027	404.0	0.8135	0.1694
0.9	418.1	335	0.9271	0.0026	434.7	0.7738	0.0396
	765.4		0.9250	0.0026	904.7	1.742	0.1821

From these tables, we can observe that the proposed two–stage procedures S_1 and S_2 work well as expected. The arguments about the comparison of efficiencies of R_1 and R_2 seems to hold for S_1 and S_2 as well. We consequently recommend the experimenter that once after taking a pilot sample, examine the value of $\widehat{p}_0$: If $\widehat{p}_0$ looks small, say $\widehat{p}_0 \leq 0.3$, proceed S_2; otherwise, even S_1 would be sufficient for such situation.

4. EXAMPLE

In the field of manufacturing of cellular phone, it is important to examine the durability of products. A cellular phone might be accidentally exposed to various situations, such as vibration and a shock, in many cases into a life. The consumers are easy to request that the weight of a cellular phone should be light, so no portions can be reinforced in order to pursue durability. Now, the experimenters would conduct an oscillating experiment to test durability of their products and to detect the lowest part of durability of the products. Here, it is considered that a cellular phone has four divided parts, that is "receiving part", "display part", "operation part" and "mouthpiece part", which are controlled by the "power supply part". If the power supply part breaks, the function of the whole cellular phone stops accordingly. We suppose that the lifetime model of a cellular phone follows the MVE distribution, where $k = 4$ and λ_0 is considered as the hazard related to failure of the "power supply part".

Let us apply a two–stage procedure proposed in Section 3 for data analysis in this situation to detect the lowest part of durability. Survival time data (x_1, x_2, x_3, x_4) are recorded in unit min. for (receiving part, display part, operation part, mouthpiece part). From the experimental side, the difference was set as $\delta^\star = 2$ and the confidence was set as $P^\star = 0.9$. We start with a pilot sample of size $m = 30$.

Table 4.1. Survival Data of Four Components (1st Stage)

(24.3, 8.9, 24.3, 7.3)	(14.7, 14.7, 14.7, 14.7)
(25.3, 4.6, 15.3, 20.5)	(27.3, 33.7, 12.9, 16.4)
(29.0, 12.3, 6.5, 21.4)	(21.5, 21.5, 21.5, 21.5)
(5.4, 5.4, 5.4, 5.4)	(22.9, 26.2, 47.6, 16.8)
(4.8, 18.0, 23.1, 26.1)	(30.1, 8.6, 36.1, 36.2)
(7.1, 7.1, 7.1, 7.1)	(16.5, 14.3, 14.4, 32.5)
(20.1, 11.6, 20.1, 10.1)	(20.9, 15.3, 13.0, 20.9)
(14.4, 12.1, 14.4, 14.4)	(16.6, 11.5, 16.6, 12.2)
(15.5, 15.5, 15.5, 15.5)	(14.4, 14.4, 14.4, 14.4)
(18.2, 11.8, 19.5, 12.2)	(23.7, 19.3, 10.1, 23.7)
(5.0, 13.5, 27.7, 27.7)	(21.2, 17.5, 21.0, 10.0)
(16.9, 16.9, 16.9, 16.9)	(23.9, 8.9, 11.8, 29.9)
(35.3, 29.7, 13.4, 35.3)	(7.1, 7.1, 6.5, 7.1)
(20.5, 20.5, 20.5, 20.5)	(28.3, 11.8, 19.0, 28.3)
(22.0, 13.9, 22.0, 22.0)	(18.4, 25.1, 29.7, 29.7)

We observe that the power supply part breaks $m_0 = 8$ times when $x_1 = x_2 = x_3 = x_4$. Then, we have $\widehat{p}_0 = 8/30 = 0.2666$. Let us proceed with Procedure S_2. From Table 2.2, we find that $u_2^\star = 1.796$ for the present case when $(k, \delta^\star) = (4, 2)$. Then, the observed value of $N^{(2)}$ is given by

$$n = \max\left\{30, \left[\frac{4.866 \times 7.666}{(1 - 0.2666) \times 3.533} u_2^{\star 2}\right] + 1\right\} = 47$$

according as (3.2). We need to do 17 additional tests and then additional data are taken as follows.

Table 4.2. Suvival Data of Four Components (2nd Stage)

(13.9, 13.9, 13.9, 13.9)	(13.9, 12.3, 15.1, 15.1)
(14.2, 13.2, 4.6, 14.2)	(4.3, 9.9, 9.9, 9.9)
(9.5, 23.7, 23.7, 18.7)	(14.6, 14.6, 14.6, 14.6)
(9.3, 9.3, 9.3, 6.5)	(17.4, 10.7, 17.4, 17.4)
(21.7, 22.6, 28.8, 28.8)	(17.1, 10.7, 25.5, 25.5)
(15.5, 9.3, 20.5, 21.8)	(7.8, 7.8, 7.8, 7.8)
(7.5, 18.2, 18.2, 18.2)	(9.8, 10.1, 13.4, 13.4)
(18.0, 11.2, 18.0, 10.1)	(14.6, 14.6, 14.6, 14.6)
(12.4, 9.7, 16.2, 25.1)	

By combing the date of both the stages, we have the overall sample means $(16.87, 14.33, 17.29, 18.13)$ for suvival time of each component. Then, the second component "display part" is detected as the worst component for durability with confidence $P^\star = 0.9$.

Remark 2. If we apply S_1 incidentally for this data set, it comes to the same conclusion to detect the display part. When p_0 is moderate or large, the experiment should be reexamined in those cases. When p_0 is small, we use S_2 based on the sample means. However, when we detect the component which has the minimum lifetime, in some cases for such purpose, the experimenter might take some censoring once after recording the minimum time along with the component number. Then, only incomplete data is available for statistician. So, it would be necessary to consider such situation as well in the next stage of this research.

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